(204)

Proc. of Workshop London, Ontario 1991

Spatially Separated Electron-Hole layers in Strong Magnetic Fields

X. M. Chen and J. J. Quinn

Department of Physics , Brown University

Providence, RI 02912

and

Department of Physics and Astronomy[†]

University of Tennessee, Knoxville, TN 37996

Studies of the collective excitation of two spatially separated electron and hole layers in strong magnetic fields indicate that the system undergoes a phase transition when the layer separation is larger than a critical value. Using the Hartree-Fock approximation, we find that this transition generates a novel excitonic density wave state, which has a lower energy than either a homogeneous exciton fluid or a double charge-density wave state. The order parameters of the state satisfy a sum rule similar to that of a charge-density wave state in a two-dimensional electron system. A possible connection between the new state and a recent experimental result is discussed.

PACS numbers: 71.35.+z, 71.45.Lr, 73.20.Dx, 73.20.Mf

[†]Present address.

I. Introduction

The nature of the ground state of the interacting two-dimensional electron system in strong magnetic fields has been studied intensively for a number of years. It is now well known that in the extreme quantum limit, at some fractional filling ($\nu = p/q \geq 1/9$, where q is an odd integer) of the lowest Landau level, the ground state is an incompressible Fermi liquid, characterized by the Laughlin wave function[1]. For $\nu < 1/9$. it has been shown that a triangular charge-density wave (CDW) state or Wigner lattice[2] is energetically more favorable than the the Laughlin fractional quantum Hall effect (QHE) state. Recently the properties of double-quantum well (DQW) systems, in which electrons are evenly distributed in each well, have received much attention, and the evolution of the ground state as the well separation is increased has been investigated both experimentally[3] and theoretically[4~6]. The steps in the Hall conductance at odd integer values of e^2/h have been observed to disappear[3] when the barrier thickness is increased. These QHE states, which correspond to the filling factor $\nu = n + 1/2$ for the average electron density in each quantum well, have been associated with the symmetric-antisymmetric (SAS) gap of the DQW. The suppression of the SAS energy gap as the well separation is increased has been suggested as the cause of the disappearance of these steps[5].

A common feature revealed in the several previous theoretical studies[4~6] of DQW's is that as the layer separations is increased the dispersion relation of the charge density excitation develops a local minimum at a wave vector on the order of the inverse of the magnetic length. This minimum becomes a soft mode when the separation reaches a critical value d_c of the order of the magnetic length. The system, therefore, undergoes a phase transition to a new state, speculated to be a CDW state. In this paper we identify this transition as the one to a novel state which we call an *excitonic charge-density wave* (ECDW) state. This new state has the properties of both an excitonic state and a normal CDW state.

The generalized system we study is the two layered electron-hole system[7], with one layer containing electrons and the other containing the equal number of holes ($\nu_e = \nu_h \equiv \nu$). The system was first introduced and intensively investigated, theoretically, by Lozovik and co-workers[8]. It can be realized either by the molecular epitaxy growth of the $InAs-AlSb-GaSb$ heterostructures[9] or by applying a strong electric field to the $GaAs - AlGaAs$ DQW's[10]. The layer width as well as the tunnelling between the two layers will be neglected throughout the paper, since they are not essential in the ECDW transition. For $\nu = 1/2$, our system is equivalent to the half-filled electron-electron DQW system studied by Fertig in Ref. 4. At small layer separations, where the interlayer Coulomb attraction is strong, electrons and holes pair together to form excitons. The excitonically condensed state of the electron-hole pairs is then the preferable ground state.[7] It has been suggestd that the system in the excitonic state may exhibit a superfluid of electron-hole pairs when an electric field is applied parallel to the layers[8]. On the other hand, if the layer separation is much larger than the average intralayer distance between neighboring particles two independent Laughlin states or triangular CDW states will give the lowest energy of the system, depending on the filling factor ν. Between these two limits, that is, when the layer separation is comparable with the intralayer particle separation, the ECDW state appears. In this novel state, both the excitonic condensation and CDW's exist. Furthermore the condensations of the excitons will occur not only at $\vec{K} = 0$ (where $\vec{K}$ is the wave vector of excitons) but also at the wave vector of the CDW's.

St-A per telecon, Larry Cooper, ONR/
Code 3143. Arlington, VA 22217

5-24-93 JK

II. Collective Excitations

We start with the general Hamiltonian of the electron-hole system in a strong magnetic field, assuming that only the first Landau levels are occupied

$$\hat{H} = \frac{1}{2L^2}\sum_{i,j,\vec{q}} V_{ij}(\vec{q})[\hat{\rho}_i(\vec{q})\hat{\rho}_j(-\vec{q}) - \delta_{ij}e^{-(ql)^2/2}\hat{\rho}_i(0)] - \mu\hat{\rho}_e(0) - \mu\hat{\rho}_h(0), \tag{1}$$

where i, j=electron or hole, μ is the chemical potential, $V_{ee}(\vec{q}) = V_{hh}(\vec{q}) = 2\pi e^2/\epsilon q$, and $V_{eh}(\vec{q}) = -2\pi e^2 \exp(-dq)/\epsilon q$. L is the linear dimension of the system, and l $(= \sqrt{\hbar c/eB})$ is the magnetic length. The spin degree of freedom of electrons and holes are frozen by the magnetic field. The particle density operators $\hat{\rho}(\vec{q})$ in the above Hamiltonian are given by

$$\hat{\rho}_e(\vec{q}) = \sum_X a^+_{X_+} a_{X_-} exp[iq_x X - (ql)^2/4], \tag{2}$$

and

$$\hat{\rho}_h(\vec{q}) = \sum_X b^+_{X_+} b_{X_-} exp[-iq_x X - (ql)^2/4], \tag{3}$$

where $X_\pm = X \pm q_y l^2/2$, and $-L/2 \le X = \frac{2\pi l^2}{L} j \le L/2$, with j being an integer. a^+_X (a_X) and b^+_X (b_X) are the creation (annihilation) operators of the electron and hole wave functions in the Landau gauge.

In the normal uniform excitonic phase, as a result of electron-hole interaction, electron-hole pairs condense into a state with zero total momentum. The order parameter in this case is just $\langle b_{-X} a_X\rangle$ or $\langle a^+_X b^+_{-X}\rangle$ which is finite and equal to $\sqrt{\nu(1-\nu)}$ for the condensed excitonic state. In order to study the evolution of the ground state as a function of the layer separation d, as well as the collective excitations of the system, we introduce a generalized operator of $a^+_X b^+_{-X}$,

$$\hat{d}^+(\vec{q}) \equiv \sum_X a^+_{X_+} b^+_{-X_-} \exp[iq_x X - (ql)^2/4], \tag{4}$$

which creates an electron-hole pair with a total momentum $\hbar\vec{q}$.

Following Anderson's treatment[11,12] of the collective excitations in superconductivity, we derived a set of extended random-phase approximation (ERPA) equations for $\langle\hat{\rho}_e(\vec{q})\rangle$, $\langle\hat{\rho}_h(\vec{q})\rangle$ and $\langle\hat{d}^+(\vec{q})\rangle$ by linearization of the equations of motion

$$i\hbar\frac{\partial\hat{\rho}}{\partial t} = [\hat{\rho}, \hat{H}],$$

with the help of the Hartree-Fock decoupling technique:

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\langle\hat{\rho}_e(\vec{q})\rangle &= i\hbar\frac{\partial}{\partial t}\langle\hat{\rho}_h(\vec{q})\rangle \\ &= \sqrt{\nu(1-\nu)}[E_{eh}(\vec{q}) - E_{eh}(0)][\langle\hat{d}(-\vec{q})\rangle - \langle\hat{d}^+(\vec{q})\rangle] \qquad (5) \end{aligned}$$

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\langle\hat{d}^+(\vec{q})\rangle &= (2\nu-1)[E_{eh}(\vec{q}) - E_{eh}(0)]\langle\hat{d}^+(\vec{q})\rangle \qquad (6) \\ &+ \sqrt{\nu(\nu-1)}\{E_{eh}(0) + E_{ee}(\vec{q}) - \frac{1}{2\pi l^2}[V_{ee}(\vec{q}) + V_{eh}(\vec{q})]e^{-(ql)^2/2}\}[\langle\hat{\rho}_e(\vec{q})\rangle + \langle\hat{\rho}_h(\vec{q})\rangle]. \end{aligned}$$

Here $E_{ee}(\vec{q})$ and $E_{eh}(\vec{q})$ are defined as follows:

$$E_{ee}(\vec{q}) = \frac{e^2}{\epsilon l}\sqrt{\frac{\pi}{2}}\exp[-\frac{(ql)^2}{4}]I_0[\frac{(ql)^2}{4}], \qquad (7)$$

and

$$E_{eh}(\vec{q}) = -\frac{e^2}{\epsilon}\int_0^\infty dt J_0(ql^2t)e^{-(tl)^2/2-td}, \qquad (8)$$

where $J_0(x)$ and $I_0(x)$ are the Bessel function and modified Bessel function of order zero respectively.

Substituting $\langle\hat{\rho}_e(\vec{q})\rangle = \rho_e e^{-i\omega(\vec{q})t}$, $\langle\hat{\rho}_h(\vec{q})\rangle = \rho_h e^{-i\omega(\vec{q})t}$, and $\langle d^+(\vec{q})\rangle = d^+ e^{-i\omega(\vec{q})t}$ into these coupled ERPA equations, we obtain the dispersion relation of the collective modes in the excitonic state[13],

$$\begin{aligned} \omega^2(\vec{q}) &= (2\nu-1)^2[E_{eh}(\vec{q}) - E_{eh}(0)]^2 \qquad (9) \\ &- 4\nu(1-\nu)[E_{eh}(\vec{q}) - E_{eh}(0)]\{E_{eh}(0) + E_{ee}(\vec{q}) - \frac{1}{2\pi l^2}[V_{ee}(\vec{q}) + V_{eh}(\vec{q})]e^{-(ql)^2/2}\}. \end{aligned}$$

We would like to remind reader that the self-exchange energy correction of electrons or holes, as well as the ladder diagram corrections have been properly included in the ERPA equations (5) and (6), consequently in the excitation energy spectrum $\omega(\vec{q})$.

At $\nu = 1/2$, the dispersion relation of Eq. (9) is exactly the same as that obtained by Fertig[4] for a half-filled electron-electron DQW, which might be expected from the electron-hole symmetry. In Fig. 1 we plot $\omega^2(q)$ versus q at $\nu = 0.45$ for several values of the layer separation d. As has been noticed by several workers[4~6] in the case of the half-filled electron-electron DQW's, $\omega^2(\vec{q})$ of Eq. (9) becomes negative at $ql \sim 1.3$ when the layer separation d is increased beyond a critical value $d_c(\nu)$. A plot of d_c as a function of ν is shown in Fig. 2; it defines the phase boundary between the uniform excitonic state and the new state (which we call the ECDW state). Also plotted in Fig. 2 are the values of q_c at which $\omega^2(q)$ first goes to negative, i.e. $\omega^2(q_c)\,|_{d=d_c}= 0$. Except for a very small filling, the critical value $q_c l$ is *insensitive to the filling factor* ν *and is approximately equal to* 1.3.

III. ECDW Ground State

The negative values of $\omega^2(q)$ for $d > d_c$ indicate the existence of static CDW distortions in the new ground state of the system. However, because of the coupling of $\langle\hat{\rho}_e(\vec{q})\rangle$, $\langle\hat{\rho}_h(\vec{q})\rangle$, and $\langle\hat{d}^+(\vec{q})\rangle$ in the ERPA equations mentioned above, the values of $\langle\hat{d}^+(\vec{q})\rangle$ at the wave vectors of the CDW's may also be finite and time independent. We therefore define the order parameters of the ECDW state

$$\begin{aligned}\Delta_{CDW}(\vec{Q}) &= \frac{2\pi l^2}{L^2}\langle\hat{\rho}_e(\vec{Q})\rangle \exp[\frac{(Ql)^2}{4}] \\ &= \frac{2\pi l^2}{L^2}\langle\hat{\rho}_h(\vec{Q})\rangle \exp[\frac{(Ql)^2}{4}], \end{aligned} \tag{10}$$

and

$$\Delta^*_{ex}(\vec{Q}) = \frac{2\pi l^2}{L^2}\langle\hat{d}^+(\vec{Q})\rangle \exp[\frac{(Ql)^2}{4}], \tag{11}$$

where $\Delta_{CDW}(0) = \nu$, and $\{\vec{Q}\}$ are the wave vectors of the ECDW. In the Hartree-Fock (HF) approximation the Hamiltonian of Eq. (1) is decoupled to

$$\begin{aligned} \hat{H} &= \sum_{X,\vec{Q}} \{U_{CDW}(\vec{Q})\Delta_{CDW}(-\vec{Q})(e^{iQ_xX}a^+_{X_+}a_{X_-} + e^{-iQ_xX}b^+_{X_+}b_{X_-}) \\ &\quad -U_{ex}(\vec{Q})[\Delta^*_{ex}(\vec{Q})e^{-iQ_xX}a_{X_+}b_{-X_-} + H.C.]\} - \mu\rho_e(0) - \mu\rho_h(0) \end{aligned} \tag{12}$$

Here $X_\pm = X \pm Q_y l^2/2$, $U_{CDW}(\vec{Q})$ is given by

$$U_{CDW}(\vec{Q}) = \frac{e^2 d}{\epsilon l^2}\delta_{\vec{Q},0} + \frac{1}{2\pi l^2}[V_{ee}(\vec{Q}) + V_{eh}(\vec{Q})]e^{-(Ql)^2/2}(1-\delta_{\vec{Q},0}) - E_{ee}(\vec{Q}), \tag{13}$$

and $U_{ex}(\vec{Q})$ is equal to $E_{eh}(\vec{Q})$ defined in Eq. (8).

The Hartree-Fock Hamiltonian (12) can be diagonalized by a series of unitary transformations. In this paper we consider only the simplest case, i.e., a unidirectional ECDW state having wave vectors $\{\vec{Q}\} = nQ_0\hat{x}$, where $n = 0, \pm1, \pm2, \cdots$. Q_0 is the fundmental periodicity of the ECDW. The Hamiltonian then reduces to,

$$\begin{aligned} \hat{H} &= \sum_X \{[E_{CDW}(X) - \mu](a^+_X a_X + b^+_X b_X) \\ &\quad + E_{ex}(X)(a_X b_{-X} + b^+_{-X}a^+_X)\} + const \end{aligned} \tag{14}$$

where

$$E_{CDW}(X) = \sum_{n=-\infty}^{\infty} U_{CDW}(nQ_0)\Delta_{CDW}(nQ_0)\cos(nQ_0X), \tag{15}$$

and

$$E_{ex}(X) = -\sum_{n=-\infty}^{\infty} U_{ex}(nQ_0)\Delta_{ex}(nQ_0)\cos(nQ_0X). \tag{16}$$

We have assumed that both Δ_{CDW} and Δ_{ex} are real quantities. $\hat{H}$ of Eq. (14) is diagonalized by a Bogolyubov transformation,

$$\begin{cases} a_X = & u_X\alpha^+_{-X} + v_X\beta^+_{-X} \\ b_X = & u_X\beta_X - v_X\alpha_X, \end{cases} \tag{17}$$

with

$$\begin{cases} u_X^2 = & \frac{1}{2}(1+\xi_X/E_X) \\ v_X^2 = & \frac{1}{2}(1-\xi_X/E_X), \end{cases} \tag{18}$$

where $\xi_X = E_{CDW}(X) - \mu$ and $E_X = \sqrt{[E_{CDW}(X)-\mu]^2 + E_{ex}^2(X)}$. The Hamiltonian of the system now becomes,

$$\hat{H} = \sum_X (-E_X \alpha_X^+ \alpha_X + E_X \beta_X^+ \beta_X) + const. \tag{19}$$

At the zero temperature only the α states are occupied. The order parameters of the ECDW ground state are, then, found to be given by:

$$\Delta_{ex}(nQ_0) = \frac{1}{2}\int_0^1 dx \cos(n\pi x) \frac{E_{ex}(\pi x/Q_0)}{\sqrt{[E_{CDW}(\pi x/Q_0)-\mu]^2 + E_{ex}^2(\pi x/Q_0)}}, \tag{20}$$

and

$$\Delta_{CDW}(nQ_0) = \frac{1}{2}\int_0^1 dx \cos(n\pi x)\left[1 - \frac{E_{CDW}(\pi x/Q_0)-\mu}{\sqrt{[E_{CDW}(\pi x/Q_0)-\mu]^2 + E_{ex}^2(\pi x/Q_0)}}\right]. \tag{21}$$

Eqs. (15), (16), (20) and (21) are a set of self-consistent equations. To find the ground state of the system for given values of ν and d, we first assume some value for Q_0, solve Eqs. (15), (16), (20) and (21) for $\Delta_{CDW}(nQ_0)$, $\Delta_{ex}(nQ_0)$ and μ, and then minimize the expectation value $E_{HF}(Q_0, d, \nu)$ of the Hartree-Fock Hamiltonian (12) with respect to Q_0. E_{HF} is given by

$$E_{HF}(Q_0, d, \nu) = \frac{L^2}{2\pi l^2}\sum_n [U_{CDW}(nQ_0) \mid \Delta_{CDW}(nQ_0) \mid^2 + U_{ex}(nQ_0) \mid \Delta_{ex}(nQ_0) \mid^2]. \tag{22}$$

Since there are infinite number of order parameters, we introduce a cut-off n_c and set $\Delta_{CDW}(nQ_0)$ and $\Delta_{ex}(nQ_0)$ to zero for $\mid n \mid > n_c$. In general, for given values of ν and d, Eqs. (15), (16), (20) and (21) have a number of solutions corresponding to different kinds of states. Among them three solutions are of particular interesting: the uniform excitonic state ($\Delta_{ex}(0) \neq 0$, $\Delta_{CDW}(nQ_0) = \Delta_{ex}(nQ_0) = 0$ for $|\ n\ | \neq 0$), the double CDW state ($\Delta_{CDW}(nQ_0) \neq 0$, $\Delta_{ex}(nQ_0) = 0$), and the ECDW state ($\Delta_{CDW}(nQ_0) \neq 0$, $\Delta_{ex}(nQ_0) \neq 0$). The self-consistent calculation has been carried out for $n_c = 8$ at several different values of the filling factor. In Fig. 3 the energy per electron-hole pair of these three states is shown as a function of the layer separation for $\nu = 0.23$. The solution for the ECDW state exists only when the layer separation is larger than some critical value, and it asymptotically approaches the solution for the double CDW state as the separation increases. For $d > d_c$, the ECDW state is energetically more favorable than both the uniform excitonic state and the double CDW state. The first three order parameters of the ground state $\Delta_{ex}(0)$, $\Delta_{CDW}(Q_0)$ and $\Delta_{ex}(Q_0)$ versus d are plotted in Fig. 4 for $\nu = 0.23$. Starting from $d = d_c$, as $\Delta_{ex}(0)$ drops rapidly, $\Delta_{ex}(Q_0)$ first increases, then decreases, and exhibits a maximum at $d \sim 1.9l$. From Eqs. (20) and (21) it can be easily shown that the order parameters of the ECDW state satisfy the sum rule

$$\sum_{\vec{Q}}[|\ \Delta_{CDW}(\vec{Q})\ |^2 + |\ \Delta_{ex}(\vec{Q})\ |^2] = \nu, \tag{23}$$

which is similar to that for a two-dimensional CDW state.[14] More interesting is that the critical layer separations d_c for the ECDW state and the corresponding wave vector Q_0, obtained from Eqs. (15), (16) and (20)~(22), are exactly the same (within 0.1%) as those values given in Fig. 1. At $\nu = 0.23$, for example, we found that $d_c = 1.34l$ and the corresponding $Q_0 l = 1.31$. We thus identify the phase transition resulting from the soft mode of the collective excitations in the uniform excitonic state as the transition leading to an ECDW state, most likely a triangular ECDW state. We believe this is also the phase transition suggested in the works by Fertig[4], Brey[6], and MacDonald *et.al.*[5]. Since in the new state the ECDW can be pinned by the impurties in the quantum wells, one should no longer expect the observation of the quantum Hall effects in the system, as the experimental results of Ref. 1 indicated.

IV. Conclusions

We have found a novel ground state of the electron-hole DQW's under strong magnetic fields, in which an excitonic condensation and crystallization coexist. The transition to such a state when the layer separation is of the order of the magnetic length is consistent with the softening of the collective modes in the uniform excitonic state. Our theory discribes the ground state of the system continuously, from an excitonic state at one end ($d=0$) to a double charge-density wave state at another ($d \rightarrow \infty$). An ECDW state can be viewed as a mixture of the two limit cases, however the transition from a uniform excitonic state to the corresponding ECDW state is well defined. The Hamiltonian (12) and the sum rule of Eq. (23) are valid for all values of d, consequently, for all of the three states.

The numerical calculation of the excitation spectrum, by Chakraborty and Pietilainen[14], for a one-half filling electron-electron double layer system at $d \sim 2.0l$, indicates that the interlayer Coulomb interaction stablizes a 'unique ground state' and opens a gap in the energy spectrum. It is very possible that this unique state is just an ECDW state we introduced here. However a definite conclusion about the connection between them can not be made before the collective excitations of the ECDW state is investigated. A study of these excitations *via* an extended random phase approximation similar to the one we used in Sec. II. is currently under way. In this paper we calculated the energy of a unidirectional ECDW state as a function of the layer separation. A more interesting problem would be the comparision of the ground state energy between a triangular ECDW and a triangular double CDW. The diagonalization of the Hamiltonian (12) for a triangular ECDW state has been carried out, and will be published elsewhere.

We would like to thank Yun Zhu, Xiaoguang Xia, and G. F. Giuliani for helpful discussions. This work is supported by U. S. Office of Naval Research under Grant No. N00014-90-J-1915.

References

[1] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[2] D. Levesque, J. J. Weis, and A. H. MacDonald, Phys. Rev. **B 30**, 1056 (1984).

[3] G. S. Boebinger, H. W. Jiang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 64, 1793 (1990).

[4] H. A. Fertig, Phys. Rev. **B 40**, 1087 (1989).

[5] A. H. MacDonald, P. M. Platzman, and G. S. Boebinger, Phys. Rev. Lett. **65**, 775 (1990).

[6] L. Brey, Phys. Rev. Lett. **65**, 903 (1990).

[7] Yu E. Lozovik and V. Yudson, Sov. Phys. JETP **44**,389 (1976); I. V. Lerner, Yu E. Lozovik and D. R. Musin, J. Phys. **C 14**, L311 (1981).

[8] D. Yoshioka and A. H. MacDonald, J. Phys. Soc. Jpn. **59**, 4211 (1990).

[9] R. Beresford, L. F. Luo, K. F. Longenbach, and W. I. Wang, Appl. Phys. Lett. **56**, 953 (1990).

[10] T. Fukuzawa, E. E. Mendez and J. M. Hong, Phys. Rev. Lett. **64**, 3066 (1990).

[11] P. W. Anderson, Phys. Rev. **112**, 1900 (1958).

[12] D. Paquet, T. M. Rice, and K. Ueda, Phys. Rev. **B 32**, 5208 (1985).

[13] The excitation spectrum of the excitonic state was first studied by Y. Kuramoto and C. Horie, Solid State Commun. **25**, 713 (1978). Their expression for $\omega^2(\vec{q})$ contains an error.

[14] D. Yoshioka and P. A. Lee, Phys. Rev. **B 27**, 4986 (1983).

[15] T. Chakraborty and P. Pietlilainen, Phys. Rev. Lett. **59**, 2784 (1987).

Figure Captions

Fig. 1. The collective charge excitation spectrum of a spataislly separated electron-hole system at $\nu = 0.45$. The layer separation d is in the unit of the magnetic length l, and $\omega(\vec{q})$ is in $e^2/\epsilon l$.

Fig. 2. The critical layer separation d_c (solid line) and the corresponding wave vector q_c (dashed line) as a function of the filling factor ν

Fig. 3. Energy per electron-hole pair in three different states at $\nu = 0.23$ versus the layer separation ; $e^2 d\nu/\epsilon l^2$ is the direct Coulomb interaction energy of the system. The vertical coordinates are in units of $e^2/\epsilon l$.

Fig. 4. Variations of the first three order parameters in the ECDW state at $\nu = 0.23$ as a function of the layer separation.